have escaped the greatest philosopher of modern times. Probably Kant was right.

W. L. Gooding

Conference Academy, Dover, Delaware

COMMUNICATION

To the Editor of the School Review :

In a book review written for the April number of the SCHOOL REVIEW, mention was made of a solution of the equation $\sqrt{2x+8} - 2\sqrt{x+5} = 2$ in which $+4$ and -4 were regarded as roots. This solution was criticised on the ground that neither $+4$ nor -4 will render the equation in its present form an identity. The review has called forth some letters of enquiry as to why these answers are objectionable and what method of solution will give the correct roots, if any roots are to be found.

There are very few even of our best algebras that give a comprehensive treatment of equivalent equations, yet a clear idea of their nature and importance is necessary to an understanding of algebra as a science. Very often, to be sure, algebra is taught as an art rather than as a science; but even as an art, algebra loses much by not considering the equivalence of equations. Some of the ordinary methods of solving algebraic problems are likely to introduce or eliminate one or more roots, and so we frequently find that the answers to problems given in some text-books in common use are incorrect, that is, when these answers are substituted for the unknown quantity, they will not satisfy the original equation. Especially is this true of equations containing surds.

It has been proved that every rational integral equation has a root; and, also, that the number of roots of every such equation corresponds to the degree of the equation. Thus, an equation of the second degree has two roots. In all the realm of real and imaginary numbers, there are to be found two quantities, and two only, that will satisfy any given equation

of the second degree. An equation of the third degree has three roots, and so on. Is it possible to apply either of these principles to equations containing surds? Some maintain that the first principle is general; that every equation has a root, real or imaginary. But in order to prove this statement they give to the radical sign a double meaning. For example, such an equation as the one mentioned above would have, according to their explanation, four distinct interpretations. They let each radical sign stand for both the negative and the positive root, and thus by combining the different values of the surd expressions in all possible ways, they have four equations instead of one. In the following discussion the radical sign, when preceded by the sign —, will stand for the negative root of the radical expression, and when preceded by the sign + (either written or understood) will stand for the positive root. Inasmuch as in algebra the sign + is understood to precede every term that is preceded by no written sign, this leaves no chance for ambiguity. Further, it does not contradict what pupils learned of the radical sign during their study of arithmetic, but is the view they naturally will take if left to themselves. Taking this point of view, the equation $\sqrt{2x+8} - 2\sqrt{x+5} = 2$ becomes easy to understand. We find that it has no root whatever.

$$\text{Let } \sqrt{2x+8} - 2\sqrt{x+5} = 2 \tag{1}$$

Squaring both numbers and transposing 2^2, we have

$$[\sqrt{2x+8} - 2\sqrt{x+5}]^2 - 2^2 = 0. \tag{2}$$

By factoring we find equation (2) equivalent to

$$[\sqrt{2x+8} - 2\sqrt{x+5} - 2]\,[\sqrt{2x+8} - 2\sqrt{x+5} + 2] = 0 \tag{3}$$

Since equation (3) is satisfied when either factor equals zero, it is equivalent to the two equations:

$$\left\{\begin{array}{ll} \sqrt{2x+8} - 2\sqrt{x+5} - 2 = 0 & (4) \\ \sqrt{2x8+} - 2\sqrt{x5+} + 2 = 0 & (5) \end{array}\right.$$

It will be seen that the root of equation (5) has been introduced by squaring. A correct solution of (2) will give all the

roots of both (4) and (5). When (2) is reduced it still contains surds, therefore we square again and have

$$9x^2+72x+144=4\ (2x+8)\ (x+5). \qquad (6)$$

Transposing and factoring as above we find equation (6) to be equivalent to the four following equations:

$$\begin{cases} \sqrt{2x+8} - 2\sqrt{x+5} = 2 & (7) \\ -\sqrt{2x+8} + 2\sqrt{x+5} = 2 & (8) \\ \sqrt{2x+8} + 2\sqrt{x+5} = 2 & (9) \\ -\sqrt{2x+8} - 2\sqrt{x+5} = 2 & (10) \end{cases}$$

The roots of equations (8) (9) and (10) have been introduced by squaring. A correct solution of equation (6) will give all the roots of each of these four equations. Now, the only roots found are + 4 and — 4. Each of these roots satisfies equation (8), but fails to satisfy any other of the four. Therefore, the original equation, as well as each of the equations (9) and (10), has no root.

The four equations to which equation (6) is equivalent are precisely the same as the four equations formed by combining the positive and negative values of the radical expressions in all possible ways. If we had started with any other of the four the result would have been the same.

S. L. Howe

Clinton Classical School, Peekskill, N. Y.